M2-S-4

INVESTIGATION OF SECONDARY STRESSES

IN BRIDGE STRUCTURES

BY

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Introduction.

The stresses in the members of a steel structure may be classed in two distinct groups, namely, "Primary Stresses" and "Secondary Stresses". Those stresses which are derived by statical analysis have come to be known as primary stresses. They are obtained upon the assumption of ideal joints without physical restraint, frictionless pins and the consequent longitudonal delivery of stress thru the member. For structures of the ordinary type, which come up in the routine practice of the engineer, these assumptions give a close approximation to the state of stress accruing, and enable the engineer to make a suitable design of the individual members; But in the actual condition of things, as is well known, the fundamental assumption upon which rests the static consideration, can seldom be more than partially fulfilled even in the best of designs. For example, in nearly all bridge structures the top chord is made as a continuous member and the present tendency is toward rigid, riveted joints thruout the truss for the purpose of stiffness. But whether the structure is riveted together at the joints or pin connected the effect of rigid joints is present to a large extent. Pin connected structures are in a large measure restrained from readjusting themselves under load by the friction moment exerted on the circumference of the pin, which is equivalent to the product of the direct stress into the radius of the pin. Actual observation has

shown in dismantling old structures that this friction is large enough to prevent motion as the eyebars were found to be cemented to the pin and had to be forcibly removed. These facts have led American engineers to believe that the restraint of the pin is nearly always sufficient to prevent readjustment of the members and, therefore, that pin connected structures are subject to distortion of their members just the same as riveted structures. This distortion of the members changes their angle of intersection and produces a moment about the joints. Mr. C.C.Schneider, in his report upon the Quebec Bridge Failure, pointed out the fact that this one cause produced a stress in some of the main members which was some forty to fifty per cent of the primary stress. Another class of stresses whose action is similar to that induced by pin friction is due to the non-intersection of the centroid axis of the truss members, at a common point. While in the best of designs, the moment due to this cause is kept very low yet it is sometimes very appreciable in special designs. A case is cited by Turneaure & Johnson (Framed Structures) of a girder of one of the elevated roads where the stress due to this cause amounted to seventy five percent of the primary stress.

The term "Secondary Stress" has been applied to stresses caused by the error of the assumptions of primary stresses, of which a few of the more important cases have been given above, and by such other causes as eccentric loads, unsymetrical members, curved members, temperature changes, yielding

supports, etc. Their consideration enters into a problem when a more critical analysis, than that of the approximate method of statics, is necessary. With the increase in size of structures, as well as the heavier loading used, these stresses become more important and recent failures have shown the inadequacy of the methods of calculation heretofore in vogue.

In this thesis, only those secondary stresses due to the distortion of the truss and the consequent non-longitudonal delivery of the stress in the members have been considered. First a number of general cases were considered in which the panel lengths and the moment of inertia of the members were varied. This was carried thru for both the Pratt type of truss and the Warren type (with and without verticles). Afterward a complete investigation was made of a one hundred sixty foot, six panel C. M. & St.P. Ry. bridge of the Pratt type. This was made by the use of influence lines. This structure was then changed into a Warren type and a comparison made of the merits of the two types. Thruout the investigation all joints, whether pin connected or not, were considered rigid except in a special case in which the effect of eccentricity of the pins in the top chord member was considered. In all cases the influence of the deformations on the alterations of leverarms, which cause the secondary stresses, was neglected. In addition, the effect of using only part of the bridge in determining stresses in one parti-

cular member was tried. The methods used were modifications of the Müller-Breslau's and Mauderla's methods as given in Grimm's textbook on Secondary Stresses.

*Manderla's Method**

Sheet No A

Figure I

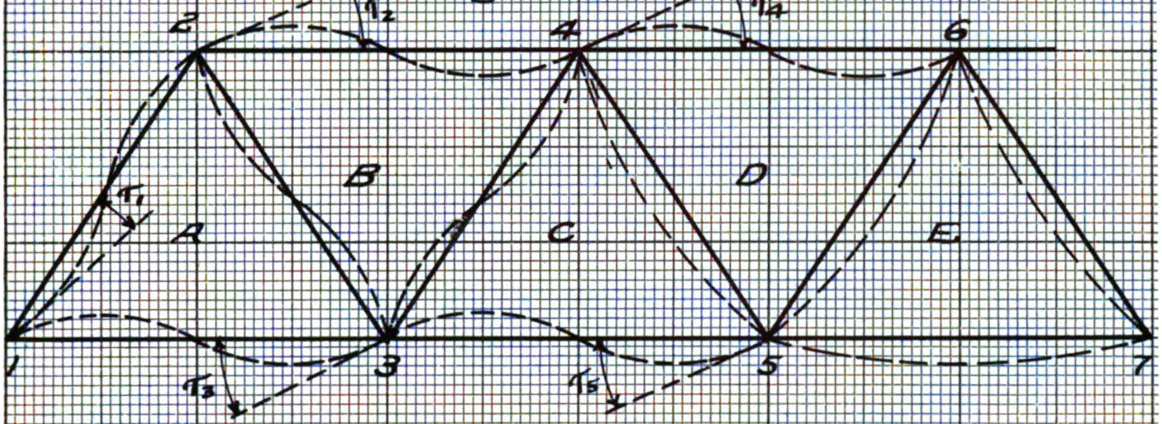
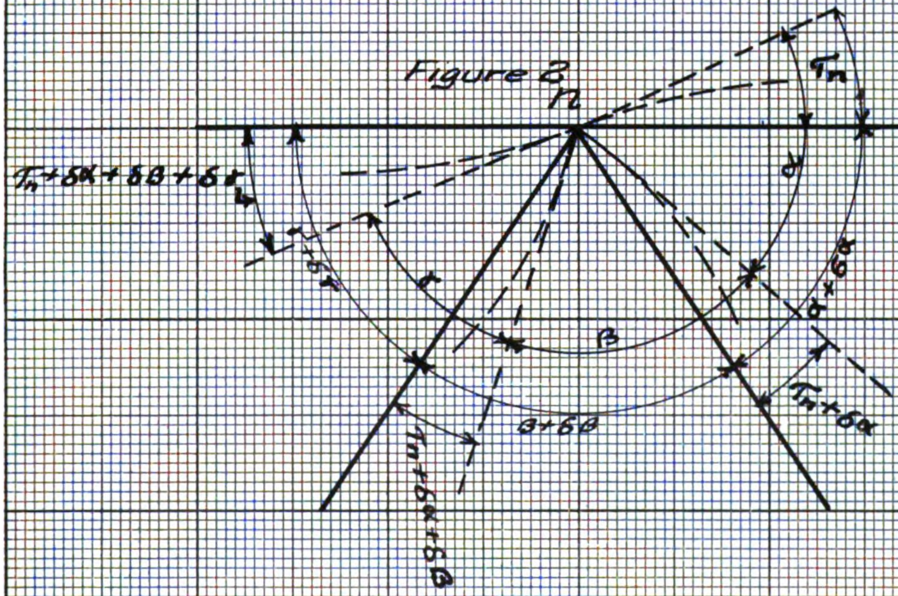


Figure 2



Note This method was used in all the general Cases

*Method modified by Dean Turneaure

In considering long trusses, especially stiffening trusses of suspension and arch bridges, it is sufficiently accurate, for purposes of investigation of the secondary stresses occuring, to assume the shear and moment constant for several panels. The make up of the members is the same, usually, for several panels, and the distortion will, therefore, be assumed constant for the several joints.

The method used in the investigation is the same as used in any truss and is briefly described below. (See sheet No. A for diagrams referred to in the demonstration.) A Warren type truss was taken, for purposes of illustration (see figure 1), although the method is applicable to any type of truss.

Assume that the truss has been loaded and distorted as shown by the dotted lines. The full lines connect the joints with one another after the distortion has taken place, and do not represent the original truss. As the members are rigidly connected at the joints the angles between them cannot change and the angles between the tangents to the distorted members are equal to the original angles of the truss.

Let α, β, and γ, (see figure 2) represent the original angles of the truss and $\alpha + \Delta\alpha$, $\beta + \Delta\beta$, and $\gamma + \Delta\gamma$ equal the angles between the lines joining the panel points after distortion. Also let τ_1, τ_2, τ_3, etc. be the angular change, between the tangent and the line joining

centers, for each member. (Then $\Delta\alpha = \tau_1 + \tau_2$, $\Delta\beta = \tau_2 + \tau_3$, $\Delta\gamma = \tau_3 + \tau_4$ etc.) For convenience τ is taken as the angular change for the member first encountered in passing around a joint in a clockwise direction and except for joint number 1 it will be the angular change of a chord member. The various changes of the tangests to the members meeting at a joint are then stated in terms of τ_m and $\Delta\alpha$, $\Delta\beta$, $\Delta\gamma$ etc. as is shown in figure 2. For this particular joint they are τ_m, $\tau_m + \Delta\alpha$, $\tau_m + \Delta\alpha + \Delta\beta$, $\tau_m + \Delta\alpha + \Delta\beta + \Delta\gamma$.

The values of $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$ are readily found from the equations $S\alpha = \frac{S_a - S_b}{E} \cot\gamma + \frac{S_a - S_c}{E} \cot\beta$ when S_a, S_b, S_c are respectively equal to the stress, in pounds per square inch, in the members opposite the angles α, β, γ.

$$\text{Now } \tau_{21} = \tfrac{1}{6}(M_{12} + 2M_{21})\frac{l}{EI}$$

$$\tau_{12} = \tfrac{1}{6}(M_{21} + 2M_{12})\frac{l}{EI}$$

when M_{12} and M_{21} are the moments of each end of the member 21 about joints 1 and 2 respectively and l, I and E are the length, moment of inertia and modulus of elasticity of member 12.

Solving and eliminating M_{21} we have,

$$M_{12} = 2E\frac{I}{l}(2\tau_{12} + \tau_{21})$$

But the sum of all the moments about a joint is equal to zero as the joint is in equilibrium. Or about joint 2 $M_{21} + M_{23} + M_{24} = 0$, and substituting for M_{21}, M_{23}, M_{24}

their values in terms of τ and omiting E, as it is a constant factor in case of an all steel structure, we have

$$\frac{I_{21}}{l_{21}}(2\tau_{21}+\tau_{12})+\frac{I_{23}}{l_{23}}(2\tau_{23}+\tau_{32})+\frac{I_{24}}{l_{24}}(2\tau_{24}+\tau_{42})=0$$

By the method explained on page 6, the values of the various τ_{n} can be expressed in terms of the reference angles τ_1 τ_2 etc., thus giving as many unknowns in the equation as there are members meeting at the joint in question. Similar equations may be written for all the joints and it will be found that there are as many equations as there are unknowns. By means of a suitable tabulation these equations can be readily solved and the values of the various τ_{n} obtained.

With the value of τ known for one member of each joint the rest may be easily computed and, substituting in the equations for "M" the bending moment of any member about the joint in question is obtained. The fibre stress is equal to $\frac{Mc}{I}$ and with M determined and c and I known the stress is readily computed.

The purpose of these general cases was to make a comparison of the secondary stresses in the Warren and the Pratt type truss; to ascertain the effect of varying the size of the web members with respect to the chord members; and to observe the effect of the length of the panels on the stresses caused. No attempt was made to detail the members and the moment of inertia and the lengths were taken in ratios. The trusses with the exception of the last three cases were investigated at the center where the moment

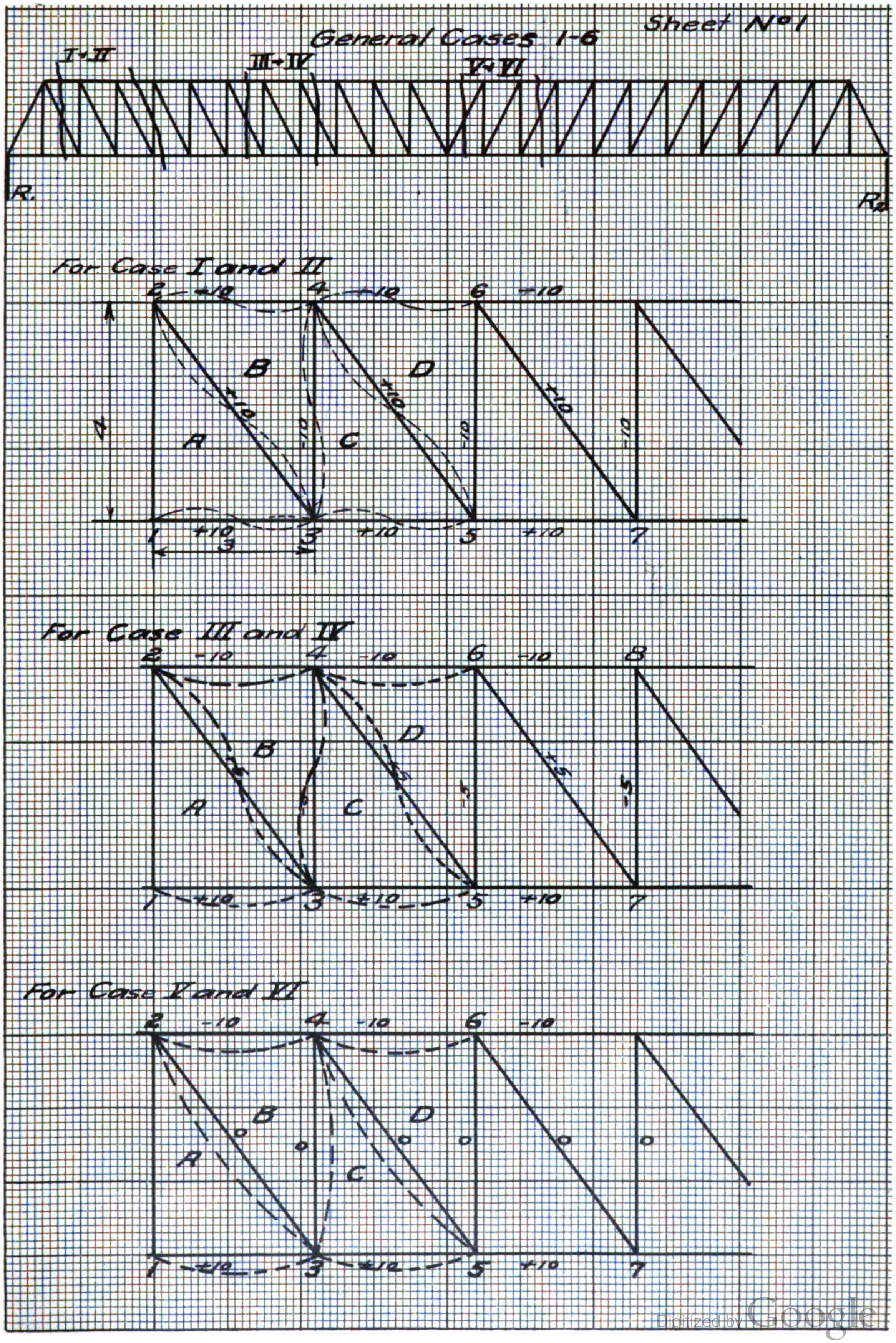
is maximum and the shear zero(or nearly so in comparison with the moment), at a mid-point where the moment is large and the shear medium, and at a point near the end where the shear is maximum and the moment relatively small.

Cases I to VI deal with the Pratt type of structure (see sheet No.1). Table 1 gives the ratio of the moments of inertia of the web members to the chord members and the stress in thousands of pounds. Sheet No. B gives a complete solution of case I. The values of I_2 I_4 and I_6 were assumed equal as also were I_1 I_3 and I_5. Table II gives the condensed results of the first six cases. It is very evident from the results that the increasing of the area

TABLE II.

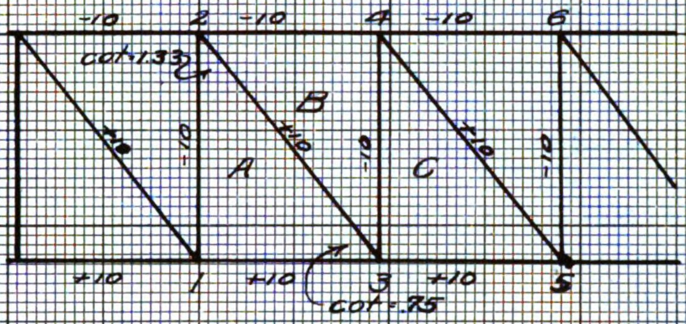
Joint	Number	Moment					
		Case I.	Case II	Case III	Case IV	Case V	Case VI
3	31	+54.2	+78.5	+36.4	+48.4	+18.4	+18.4
	32	-23.7	-39.6	-10.4	-16.9	+2.9	+5.8
	34	-44.6	-79.4	-22.4	-39.9	-0.1	-0.2
	35	+14.3	+39.6	-3.4	+8.5	-21.6	-21.6
4.	46	+17.1	+45.0	-0.9	+13.6	-18.4	-18.4
	45	-29.6	-51.0	-16.3	-28.5	-2.9	-5.8
	43	-44.4	-78.8	-22.2	-39.3	+0.1	+0.2
	42	+57.0	+84.9	+39.0	+53.5	+21.6	+21.6

sufficient to double the moment of inertia, increases the secondary stress in nearly the same proportion. This ratio keeps nearly constant all the way across the structure.


General Cases 1-6
Sheet No 1
I+II
III+IV
V+VI
R.
R2
For Case I and II
2
4
6
-10
-10
-10
B
D
A
C
+10
+10
+10
-10
-10
-10
4
1
3
5
7
+10
+10
+10
3
For Case III and IV
2
4
6
8
-10
-10
-10
B
D
A
C
+5
-5
-5
1
3
5
7
+10
+10
+10
For Case V and VI
2
4
6
-10
-10
-10
B
D
A
C
0
0
0
0
0
0
1
3
5
7
+10
+10
+10

General Case I Pratt Type

$I_{chords} = 4$
$I_{webs} = 1$
stress in chords = 10
stress " webs = 10



Triangle	Angle	δL
	231	-.2667
A	312	+.2667
	123	0
	243	+.4167
B	342	-.1500
	324	-.2667

Assume $T_2 = T_4 = T_6$ and $T_1 = T_3 = T_5$

Joint	L	δL	ΣδL	Mem	I/l = k	kΣδL	T_n+ΣδL	$2(2T_{mn}+T_{nm})$	Moment
			0	31	1.33	0	+11.80	+40.87	+54.2
	132	-.2667	-.2667	32	.20	-.534	-14.87	-118.54	-23.7
3	234	-.1500	-.4167	34	.25	-10.42	-29.87	-178.52	-44.6
	435	+.2667	-.1500	35	1.33	-19.95	-3.20	+10.80	+14.3
					3.11	-35.71			
			0	46	1.33	0	-2.86	+12.84	+17.1
	645	-.2667	-.2667	45	.20	-.534	-29.53	-147.86	-29.6
4	543	0	-.2667	43	.25	-.667	-29.53	-177.86	-44.4
	342	+.4167	+.1500	42	1.33	+19.95	+12.14	+42.84	+57.0
					3.11	+7.95			

Equations of equilibrium about joints 3 + 4

(3) $6.22T_3 + 1.33T_3 + .20T_4 + .25T_4 + 1.33T_3 - 103.38 = 0$

$8.88T_3 + .45T_4 - 103.38 = 0$

(4) $6.22T_4 + 1.33T_4 + .20T_3 + .25T_3 + 1.33T_4 + 20.04 = 0$

$8.88T_4 + .45T_3 + 20.04 = 0$

Solving for T_3 and T_4 we have

$T_4 = -2.86$ $T_3 = +11.80$

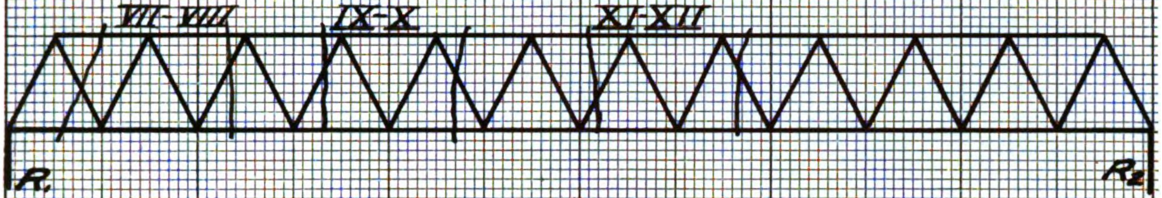
TABLE I.

General Case.

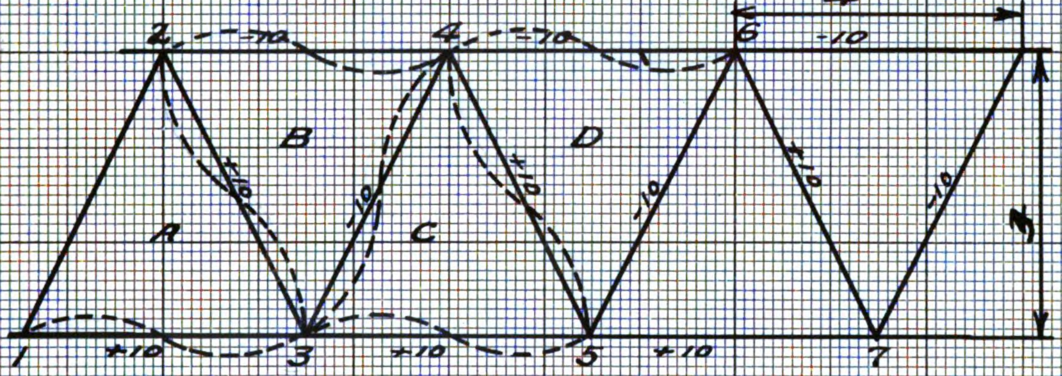
Case.	Chord Stress	Web Stress	I chord	I web.	Type.
1	10	10	4	1	Pratt
2	10	10	4	2	"
3	10	5	4	1	"
4	10	5	4	2	"
5	10	0	4	1	"
6	10	0	4	2	"
7	10	10	4	1	Warren
8	10	10	4	2	"
9	10	5	4	1	"
10	10	5	4	2	"
11	10	0	4	1	"
12	10	0	4	2	"
13	10	10	4	2	Without Verticals. Warren 60°
14	10	10	4	2	Warren 45
15	10	10	4	2	Warren 45° with verticals.

Sheet No 2

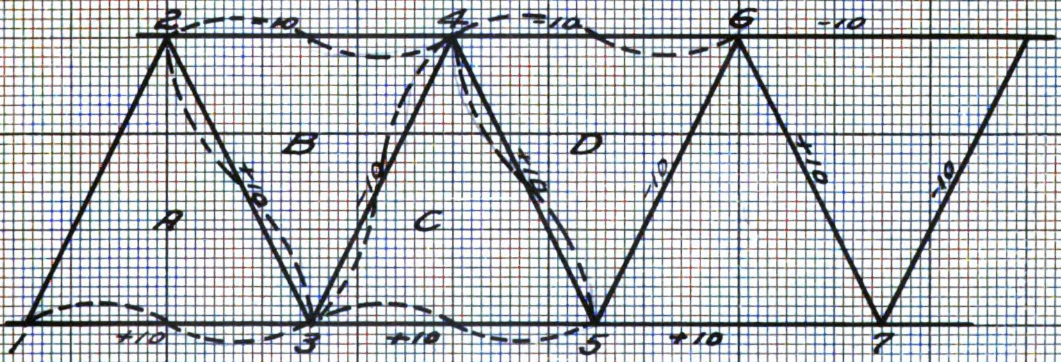
General Cases 7-12



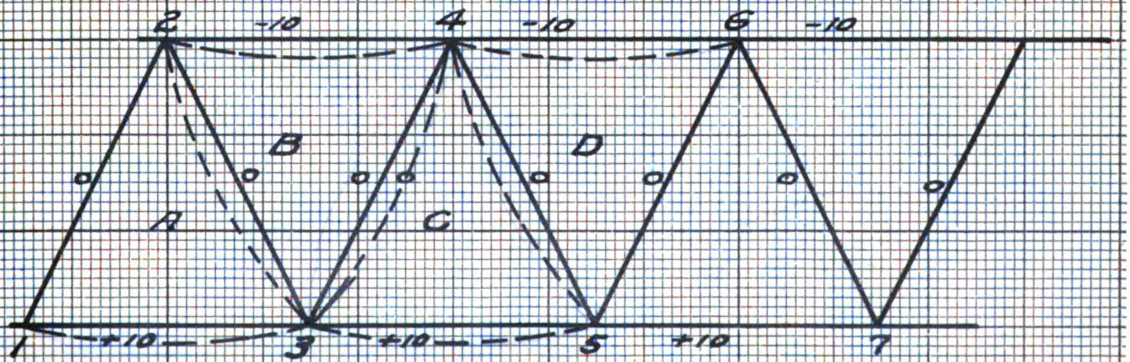
For Cases VII and VIII



For Cases IX and X



For Cases XI and XII



A comparison of the stresses for Cases I, III and V gives some idea of the distribution of secondary stresses in a bridge. At the center their action is very small and may be neglected while near the abutments the amount is considerable. Roughly the secondary stress near the points of support is from two to four times what it is at the center for chord members, and from four to ten times for web members.

Vases VII to XII deal with the Warren type of structure without verticals(see sheet No.2). Table I gives the ratio of the moments of inertia of web members to chord members and stress in thousands of pounds per square inch. The solution is exactly similar to that of the Pratt type. The ratio of the height of truss to panel length was as three is to four. The values of τ_2 τ_4 and τ_6 and τ_1 τ_3 and τ_5 were assumed equal thus simplifying the equation. Table III. gives the results for the six cases.

TABLE III.

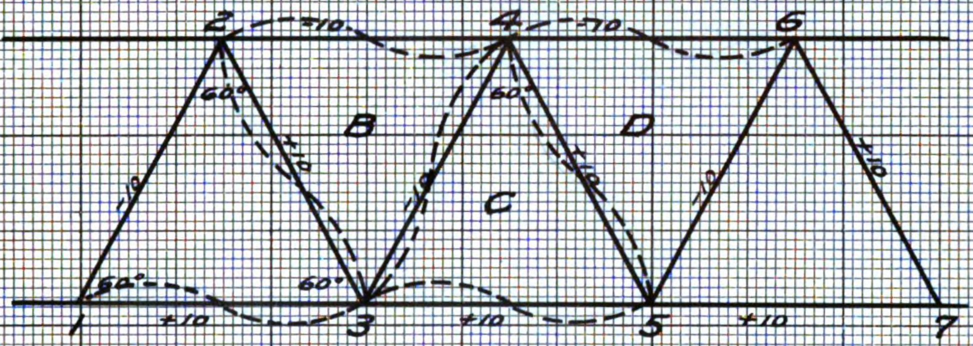
Moments

Joint	Members	Case VII	Case VIII	Case IX	Case X	Case XI	Case XII.
3	31	+39.2	+53.2	+23.0	+35.3	+16.7	+16.7
	32	-20.5	-34.0	-18.2	-12.0	+2.3	+4.6
	34	-25.1	-42.0	-27.4	-25.4	-2.3	-4.6
	35	+5.9	+21.7	-5.4	+2.1	-16.7	-16.7
4	46	+6.5	+20.8	-5.2	+1.2	-16.7	-16.7
	45	-25.1	-42.7	-27.4	-19.1	-2.3	-4.6
	43	-20.4	-34.2	-17.10	-18.10	+2.3	+4.6
	42	+39.9	+54.2	+28.2	+34.7	+16.7	+16.7

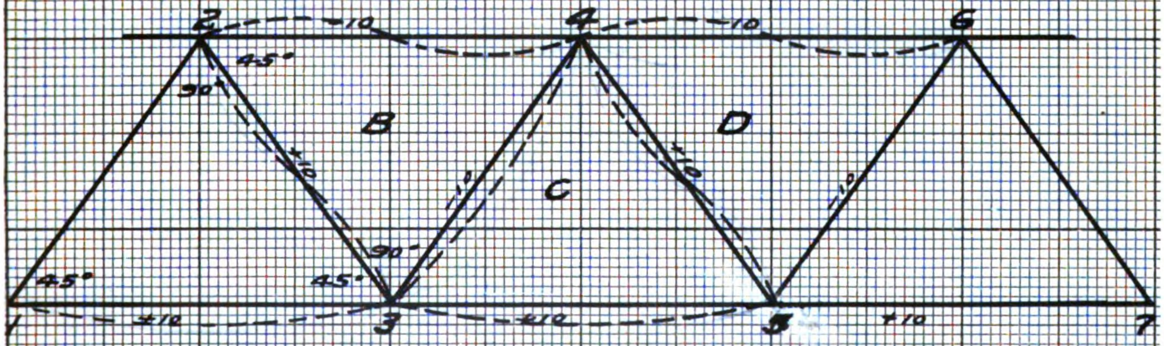
Sheet No. 3.

General Cases 13-15

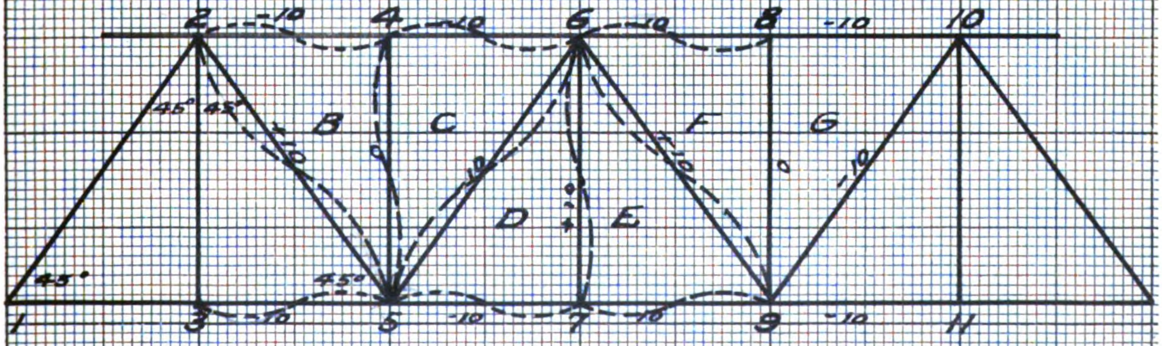
For Case XIII



For Case XIV



For Case XV



Just as in the case of the Pratt type, it is found that increasing the moment inertia increases the secondary stress although the increase does not seem to be so great with the Warren type. Here again it is found that the largest stress occurs near the supports and decreases to about one half to one fourth the amount at the center. A comparison of the two types shows very clearly the advantage of the Warren type over the Pratt type as far as the existing secondary stresses is concerned.

Cases XIII to XV deal with the effect of changing the length of panels in the Warren type and also the effect of vertical members on the amount of secondary stress. See sheet No.3 for details of the trusses. In case XIII an equilateral triangular truss was taken; in case XIV a truss whose web members made an angle of 45° to chord members; in case XV, a truss similar to that of XIV except vertical members were put in. The ratio of chord members to web members was kept constant. Table IV gives the results of case XIII and XIV in connection with VII. Sheet No. C gives the solution of case XV. In this case it was necessary to use four joints in determining the moments as the joints are not all symetrical. In this case $\tau_2 = \tau_6$, $\tau_4 = \tau_8$ and $\tau_3 = \tau_7$, $\tau_1 = \tau_5$.

Sheet N° C

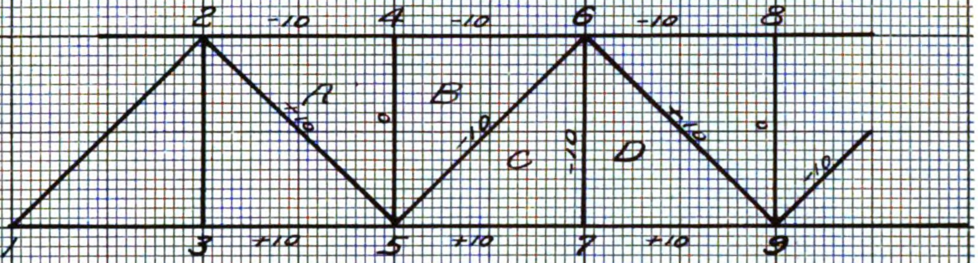
General Case XV

Warren Type With Verticals

$I_{chords} = 4$

$I_{webs} = 2$

$I_{vert} = 1$



Triangle	Angle	δL	Triangle	Angle	δL
	252	-200		657	+200
A	542	+300	C	576	-400
	425	-100		765	+200
	456	0		679	0
B	564	+100	D	796	0
	645	-100		967	0

Assume $\tau_2 = \tau_6 = \tau_{10}$

$\tau_4 = \tau_8 = \tau_{12}$

$\tau_3 = \tau_7 = \tau_{11}$

$\tau_5 = \tau_9 = \tau_{13}$

Solving for τ_5, as in the previous case

$\tau_4 = -4.1$, $\tau_5 = 5.4$ $\tau_6 = -6.5$ $\tau_7 = 20.7$

Joint	Angle	δL	$\Sigma\delta L$	Mem	$I/l = k$	$k\Sigma\delta L$	$\tau + \Sigma\delta L$	$2(2\tau_{mn} + \tau_{nm})$	Moment
			0	46	2.000	0	-4.1	+10.6	+21.2
4	645	-10.0	-10.0	45	.500	-5.0	-14.1	-85.6	-42.8
	542	+30.0	-20.0	42	2.000	+40.0	+15.9	+50.6	+101.2
					4.500	+35.0			
			0	53	2.000	0	+5.4	-17.0	-34.0
	352	0	0	52	.707	0	+5.4	-11.4	-8.1
5	254	-20.0	-20.0	54	.500	-10.0	-14.6	-86.6	-43.3
	456	0	-20.0	56	.707	-14.2	+5.4	+28.6	+20.2
	657	+20.0	0	57	2.000	0	+25.4	+143.0	+286.0
					5.914	-24.2			
			0	68	2.000	0	-6.5	+5.8	+11.6
	869	-10.0	-10.0	69	.707	-7.1	-16.5	-55.4	-39.2
6	967	0	-10.0	67	.500	-5.0	-16.5	-104.6	-52.3
	765	+20.0	+10.0	65	.707	+7.1	+3.5	+25.6	+18.2
	564	+10.0	+20.0	64	2.000	+40.0	+13.5	+45.8	+91.6
					5.914	+35.0			
			0	75	2.000	0	+20.7	+133.6	+267.2
7	576	-40.0	-40.0	76	.500	-20.0	-19.3	-110.2	-55.1
	679	0	-40.0	79	2.000	-80.0	-19.3	-66.4	-132.8
					4.500	-100.0			

TABLE IV.

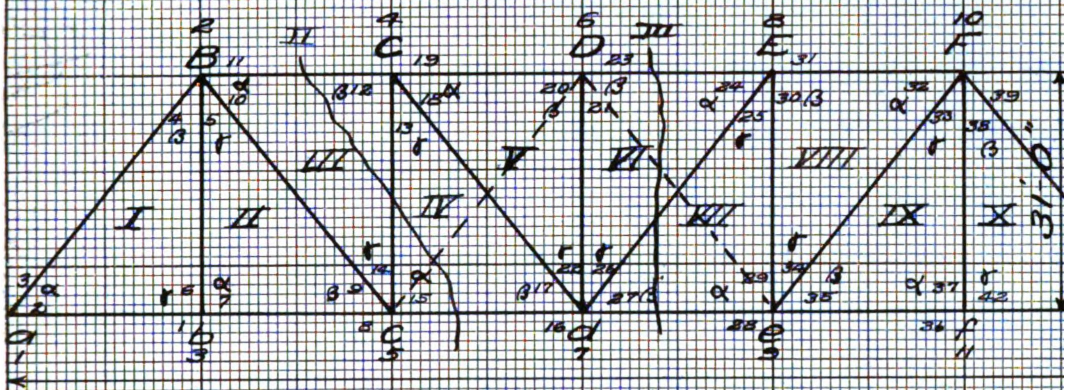
Joint	Members	Case VII	I/l	Case XIII	I/l	Case XIV	I/l
3	31	+53.2	+1.00	+50.7	+1.00	+58.6	1.00
	32	-34.0	..55	-30.0	.44	-4.8	.71
	34	-42.0	.55	-36.6	.44	-33.1	.71
	35	+21.7	1.00	+0.6	1.00	-21.4	1.00
4	46	+20.8	1.00	+7.8	1.00	+22.4	1.00
	45	-42.7	.55	-38.4	.44	-22.7	.71
	43	-34.2	.55	-26.7	.44	+5.6	.71
	42	+54.2	1.00	+54.0	1.00	+102.4	1.00

From the results it is evident that increasing the panel length decreases the secondary stresses in the web members but the stress in the chord members does not seem to be effected by change in length. In the case of member 42 especially the stress in case XIVshows a very material increase. A comparison of case XV with case XIV shows that with the addition of verticals the stresses are very greatly increased. This is due to the increase of the ratio of I/l due to short panels.

GENERAL DRAWING

160 FT 6 PANEL PRATT

TRUSS



6 @ 26'-8" = 160'-0

In order to make a comparison between the primary stresses and the secondary stresses occuring in a bridge structure, a 160 foot six panel C.M.& ST.P. Railway bridge of the Pratt type was investigated. The primary stress and details of the members were taken from drawings furnished by that company. Sheet No. "D" gives a general drawing of the bridge. The method used was a modification of the "Müller-Breslau" method. In order to more clearly show the effect of various loads on the amount of secondary stress it was decided to use influence lines. This necessitated the computing of stresses for loads at each panel point but the work was shortened by using such loads as would not change the primary stresses in most of the members. Table V gives the areas, lengths, moment of inertia and distance to extreme fibre of the member of the truss. Table VI gives the primary stress for loads placed at "c", "d", and "e".

A complete demonstration of method used will be found in Grimm's book on secondary stresses, and the notes given to the class on secondary stresses under Dean Turneaure. Only an outline of the method will be given here.

The values of $\Delta\alpha$, $\Delta\beta$ and $\Delta\gamma$ are determined in the same way as given on Page 6. As these depend on the primary stress only they were not computed for a change of load unless a change of stress occured. Referring to sheet No. A.

$$\tau_2 = \frac{1}{6}(m_1 + 2m_2)\frac{l}{EI}$$

$$\theta_3 = \tfrac{1}{6}(2M_3 + M_4)\frac{l_2}{EI_2}$$

but

$$\theta_2 + \theta_3 = \Delta\alpha$$
$$\theta_4 + \theta_5 = \Delta\beta$$
$$\theta_1 + \theta_6 = \Delta\gamma$$

and by substituting

$$(M_1 + 2M_2)\frac{l_1}{I_1} + (2M_3 + M_4)\frac{l_2}{I_2} = 6E\Delta\alpha$$
$$(M_3 + 2M_4)\frac{l_2}{I_2} + (2M_5 + M_6)\frac{l_3}{I_3} = 6E\Delta\beta$$
$$(M_5 + 2M_6)\frac{l_3}{I_3} + (2M_1 + M_2)\frac{l_1}{I_1} = 6E\Delta\gamma$$

For the sake of convenience let $U = M\frac{l}{I}$

The equations simplify to the form

$$6E\Delta\alpha = U_1 + 2U_2 + 2U_3 + U_4$$
$$6E\Delta\beta = U_3 + 2U_4 + 2U_5 + U_6$$
$$6E\Delta\gamma = U_5 + 2U_6 + 2U_1 + U_2$$

But $U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_5 \quad U_6 = 0$

Solving for values of U_4, U_5, U_6 we have

$$U_4 = 6E\Delta\alpha - U_1 - 2U_2 + 2U_3$$
$$U_5 = 6E\Delta\beta + U_1 + U_2 - U_4$$
$$U_6 = 6E\Delta\gamma - U_1 + U_3 + U_4$$

This form is applicable to any triangle by changing the subscripts. By proper tablation $U_4 \ldots U_{40}$ can be found in terms of U_1 and U_2 which are readily solved and the numerical values obtained.

But $U = M\frac{l}{I}$, $\quad M = \frac{UI}{l}$ and $M = \frac{SI}{c}$, $\quad S = \frac{Mc}{I} = \frac{UIc}{Il} = U\frac{c}{l}$

Table VII gives the solution of the secondary stresses for a load of 1000# at "e". In an exactly similar way the

TABLE V.

Member.	Area Sq.in.	Length Feet	Length Inches.	Moment Inertia Ix	Distance from neut. axis Top C	Distance from neut. axis Bot. C'
aB 1&2	53.49	40.92	491	4490.0	9.54	14.08
BC 11&12	42.35	26.67	320	3978.0	9.19	14.43
CD 19&20	42.35	26.67	320	3978.0	9.19	14.43
ab 3&4	29.449	26.67	320	1218.0	9.12	9.12
bc 9&10	29.449	26.67	320	1218.0	9.12	9.12
cd 15&16	45.489	26.67	320	1907.0	9.12	9.12
Bc 7&8	26.00	40.92	491	138.7	4.00	4.00
Cd 17&18	20.58	40.92	491	358.6	6.00	6.00
Bb 5&6	16.009	31.00	372	94.8	5.375	5.375
Cc 13&14	26.48	31.00	372	750.2	7.50	7.50
Dd 21&22	14.70	31.00	372	288.00	6.00	6.00

Member	I/l	c/l
aB 1&2	9.151	.01943
BC 11&12	12.431	.02870
CD 19&20	12.431	.02870
ab 3&4	3.805	.02850
bc 9&10	3.805	.02850
cd 15&16	5.959	.02850
Bc 7&8	0.283	.00815
Cd 17&18	0.731	.01220
Bb 5&6	0.255	.0144
Cc 13&14	2.016	.0144
Dd 21&22	0.744	.01862

TABLE VI.

Stress Table.

Member	Area	Load 1000# at d.		Load 1500# at e.		Load 3000# at f.	
	sq.in.	Stress.	#sq.in.	Stress.	#sq.in.	Stress.	#sq.in.
AB	58.49	+658	-11.2	+658	-11.2	+650	-11.2
BC	52.35	+860	-16.4	+860	-16.4	+860	-16.4
CD	52.35	+1290	-24.7	+1290	-24.7	+1290	-24.7
DE	52.35	+1290	-24.7	+1290	-24.7	+1290	-24.7
EF	52.35	+860	-16.4	+1720	-32.8	+1720	-32.8
ab	29.44	-430	+14.6	-430	+14.6	-430	+14.6
bc	29.44	-430	+14.6	-430	+14.6	-430	+14.6
cd	45.48	-860	+18.9	-860	+18.9	-860	+18.9
de	45.48	-860	+18.9	-1720	+37.8	-1720	+37.8
ef	29.44	-430	+14.6	-860	+29.2	-2150	+73.0
fg	29.44	-430	+14.6	-860	+29.2	-2150	+73.0
Bc	26.00	-658	+25.3	-658	+25.3	-658	+25.3
Cd	20.55	-658	+32.0	-658	+32.0	-658	+32.0
Ed	20.55	-658	+32.0	-658	-32.0	+658	+32.0
Fe	26.00	-658	+25.3	-1320	+50.6	+658	+25.3
Bb	16.00	000	000	000	000	000	000
Cc	26.48	+500	-18.9	+500	-18.9	+500	-18.9
Dd	14.70	000	000	000	000	000	000
Ee	26.48	+500	-18.9	-500	+18.9	+2150	+81.3
Ff	16.00	000	000	000	000	-3000	-187.5
Fg	58.49	+658	-11.2	+1320	-22.4	+3300	-56.4

TABLE VII.

$6E\Delta\alpha$ $\Delta\sigma, \Delta\sigma$	Equations of $U_1 \ldots U_{42}$	I/1	Absolute	Coef. U_1	Coef. U_2	Numerical Val. of U.
	$U_1 = U_1$	3.805		+1.000		-6.5
+78.0	$U_2 = U_2$	3.805			+1.000	-44.8
	$U_3 = -.416\,U_2$	9.151			-.416	+18.7
+133.2	$U_4 = -6E\Delta\alpha + U_1 - 2U_2 + 2U_3$	9.151	-78.0	+1.000	- 2.832	+68.5
-211.2	$U_5 = 6E\Delta\sigma + U_1 - U_2 + U_4$	0.255	+55.2	+2.000	-3.832	+195.7
	$U_6 = -6E\Delta\sigma + U_1 - U_3 + U_4$	0.255	+133.2	+2.000	-2.416	+184.1
+231.6	$U_7 = -U_1 - .06696\,U_6$	3.805	-8.9	-1.134	+0.162	-5.9
	$U_8 = 6E\Delta\alpha + U_5 - 2U_6 + 2U_7$	3.805	+2.6	-4.268	+1.324	-29.8
-1764	$U_9 = -6E\Delta\sigma + U_5 - U_6 + U_8$	0.283	+101.0	-4.268	-0.092	+99.4
-552	$U_{10} = 6E\Delta\sigma + U_5 - U_7 + U_8$	0.283	+11.5	-1.134	-2.670	+134.8
-307.2	$U_{11} = -.7362U_4 - .0205U_5 - .0227U_{10}$	12.431	+56.0	-.751	+2.224	-575
+522 .6	$U_{12} = -6E\Delta\alpha + U_9 - 2U_{10} + 2U_{11}$	12.431	+497.2	-3.502	+9.696	-80.7
-2154	$U_{13} = 6E\Delta\sigma + U_9 - U_{10} + U_{12}$	2.016	+1109.3	-6.636	+12.274	+232.3
	$U_{14} = -6E\Delta\sigma + U_9 - U_{11} + U_{12}$	2.016	+757.6	-7.019	+7.380	+219.8

TABLE VII CONT'D.

	Equations of $U_1 \ldots U_{42}$	I/1	Absolute	Coef. U_1	Coef. U_2	Numerical val. of U
+422.6	$U_{15} = -338U - .0474U_{14} - 6368U_8$	5.959	-262.8	+5.303	-3.338	-60.1
	$U_{16} = 6E\Delta\alpha + U_{13} - 2U_{14} + 2U_{15}$	5.959	-509.1	+18.008	-9.162	-46.0
-354.6	$U_{17} = -6E\Delta\theta + U_{13} - U_{14} + U_{16}$	0.731	+197.2	+18.391	-4.268	+202.8
-67.8	$U_{18} = 6E\Delta\phi + U_{13} - U_{15} + U_{16}$	0.731	+795.2	+6.069	+6.450	+201.0
-223.2	$U_{19} = -.0588U_{18} - .1622U_{13} - U_{12}$	12.431	-723.9	+4.223	-12.067	+31.1
	$U_{20} = -6E\Delta\alpha + U_{17} - 2U_{18} + 2U_{19}$	12.431	-2617.8	+14.699	-41.302	+12.0
+515.4	$U_{21} = 6E\Delta\theta + U_{17} - U_{18} + U_{20}$	0.774	-2700.4	+27.022	-52.021	+357.2
-292.2	$U_{22} = -6E\Delta\phi + U_{17} - U_{19} + U_{20}$	0.774	-1404.5	+28.868	-33.504	+378.4
	$U_{42} = U_{42}$	3.805		+1.000		-67.0
+157.2	$U_{41} = U_{41}$	3.805			+1.000	+41.5
+267.0	$U_{40} = -.416\, U_{41}$	9.151			-.416	-17.3
	$U_{39} = 6E\Delta\alpha + U_{42} - 2U_{41} + 2U_{40}$	9.151	+157.2	+1.000	-2.832	-79.6
-424.2	$U_{38} = -6E\Delta\theta + U_{42} - U_{41} + U_{39}$	0.255	-109.8	+2.000	-3.832	-366.0

TABLE VII CONT'D.

	Equations of $U_1 \ldots U_{42}$	I/1	Absolute	Coef. U_1	Coef. U_2	Numerical val. of U
	$U_{37} = 6E\Delta\sigma + U_{42} - U_{40} + U_{39}$	0.255	-2670	+2.000	-2.416	-412.2
+463.8	$U_{36} = -.06696U_{37} - U_{42}$	3.805	+17.9	-1.134	+0.162	+94.5
-353.4	$U_{35} = -6E\Delta\alpha + U_{38} - 2U_{37} + 2U_{36}$	3.805	-3.8	-4.268	+1.324	+338.3
-110.4	$U_{34} = 6E\Delta\beta + U_{38} - U_{37} + U_{35}$	0.283	-200.0	-4.268	-0.092	+148.9
	$U_{33} = -6E\Delta\sigma + U_{38} - U_{36} + U_{35}$	0.283	-21.1	-1.134	-2.670	-48.9
	$U_{32} = -.7362U_{39} - .0205U_{38} - .0227U_{35}$	12.431	-112.9	-.751	+2.224	+67.3
-221.4						
	$U_{31} = 6E\Delta\alpha + U_{34} - 2U_{33} + 2U_{32}$	12.431	-605.0	-3.502	+9.696	+233.5
+652.2						
	$U_{30} = -6E\Delta\beta + U_{34} - U_{33} + U_{31}$	2.016	-1436.1	-6.636	+12.274	-3.8
-430.8						
	$U_{29} = 6E\Delta\sigma + U_{34} - U_{32} + U_{31}$	2.016	-1122.9	-7.019	+.7.380	+37.7
	$U_{28} = -.3338U_{29} - .0474U_{34} - .6368U_{35}$	5.959	+391.7	+5.305	-3.338	-232.9
-715.2						
	$U_{27} = -6E\Delta\alpha + U_{30} - 2U_{29} + 2U_{28}$	5.959	+2308.3	+18.008	-9.162	-47.7
	$U_{26} = 6E\Delta\beta + U_{30} - U_{29} + U_{27}$	0.731	+2350.3	+18.391	-4.268	+157.7
+360.0						
	$U_{25} = -6E\Delta\sigma + U_{30} - U_{28} + U_{27}$	0.731	+120.5	+6.069	+6.450	-58.8

TABLE VII CONT'D.

	Equations of U ...U	I/1	Absolute	Coef.U_{42}	Coef.U_{41}	Numerical Val.of U
	$U_{24} = -U_{31} - .1622U_{30} - .0588U_{25}$	12.431	+830.9	+4.223	-41.302	-34.8
+223.2						
	$U_{23} = 6E\Delta\alpha + U_{26} - 2U_{25} + 2U_{24}$	12.431	+3994.3	+14.699	-41.302	-34.8
-261.6	U =					
	$U_{21} = -6E\Delta\sigma + U_{26} - U_{25} + U_{23}$	0.774	+6485.7	+27.022	-52.021	+356.3
+38.4						
	$U = 6E\Delta r + U \quad -U + U$	0.774	+5552.1	+28.868	-38.504	+378.0

TABLE VIII.

Member	Member	DIRECT STRESS LOADS AT b	c	d	e	f	SECONDARY STRESS b	c	d	e	f
ab	1	+24.3	+19.5	+14.6	+9.8	+5.0	+15.1	+1.95	+.31	+.21	+.10
	2	+24.3	+19.5	+14.6	+9.8	+5.0	+10.0	+1.16	+1.89	+1.26	+.63
aB	3	-18.8	-15.0	-11.2	-7.5	-3.8	-4.18	-.48	-.79	-.53	-.26
	4	-18.8	-15.0	-11.2	-7.5	-3.8	+1.96	-2.46	-2.87	-1.91	-.95
Bb	5	-62.5	0	0	0	0	+.82	-5.12	-4.16	-2.76	-1.37
	6	-62.5	0	0	0	0	-.20	-5.92	-3.91	-2.60	-1.30
bc	7	+24.3	+19.5	+14.6	+9.8	+5.0	+15.00	-2.50	-.20	-.12	-.07
	8	+24.3	+19.5	+14.6	+9.8	+5.0	+8.60	-9.55	-1.08	-.72	-.35
cB	9	+8.1	+33.7	+25.3	+16.2	+8.1	-2.11	-1.22	+1.25	+.83	+.42
	10	+8.1	+33.7	+25.3	+16.2	+8.1	-.89	+.36	+1.63	+1.08	+.55
BC	11	-10.9	-21.8	-16.4	-11.9	-5.4	+3.56	-1.92	-2.40	-1.60	-.80
	12	-10.9	-21.8	-16.4	-11.9	-5.4	+.70	-6.60	-3.12	-2.11	1.04
Cc	13	+29.1	+12.6	-18.9	-12.6	-6.3	-4.61	+.10	+7.52	+4.89	+2.45
	14	+29.1	+12.6	-18.9	-12.6	-6.3	-6.30	-.62	+6.90	+4.60	+2.31
cd	15	+12.6	+25.2	+18.9	+12.6	+6.3	+2.17	+1.04	+2.82	+1.88	+.94
	16	+12.6	+25 .2	+18.9	+12.6	+6.3	-.35	-1.82	+2.82	+1.88	+.94

TABLE VIII CONT'D.

Member	Member	DIRECT STRESS LOADS AT b	c	d	e	f	SECONDARY STRESS b	c	d	e	f
dc	17	+10.6	+21.3	+32.0	+21.3	+10.6	-.24	-2.48	+3.39	+2.25	+1.13
	18	+10.6	+21.3	+32.0	+21.3	+10.6	-.26	+.87	+3.61	+2.41	+1.21
CD	19	-8.2	-16.5	-24.7	-16.5	-8.3	1.09	-10.20	-1.50	-.99	-.50
	20	-8.2	-16.5	-24.7	-16.5	-8.3	+.15	-1.01	-.82	-.54	-.27
Dd	21	0	0	0	0	0	-1.69	-6.70	+8.70	+5.80	+2.88
	22	0	0	0	0	0	1.11	-1.70	-9.43	-6.30	-3.15

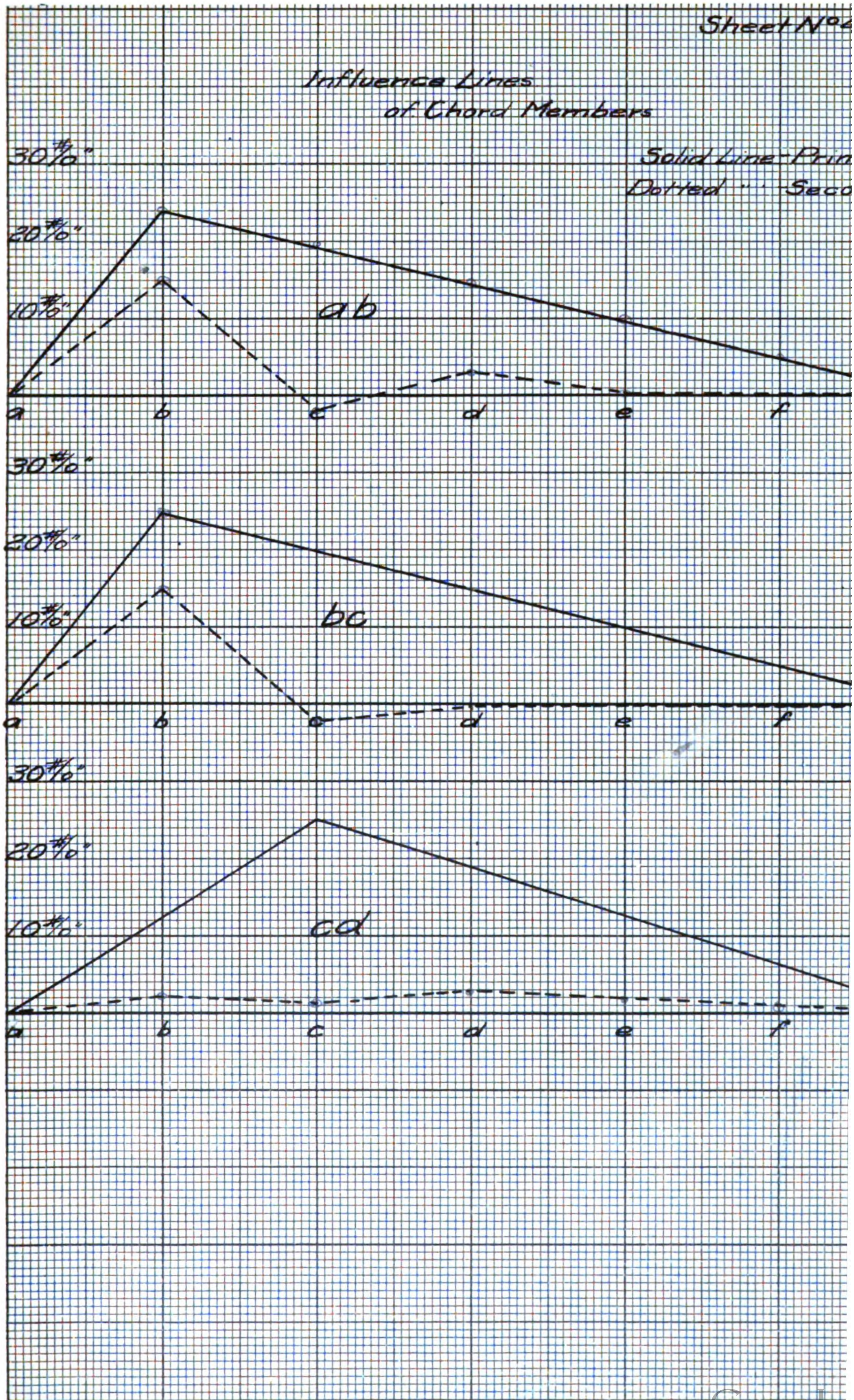
stresses were determined for loads at b, c, d, and f, and are given in table VIII.

Table IX gives the combination of table VIII for live and dead loads and the percent of secondary stress to primary stress. Sheets number 4, 5, 6, 7 represent the influence lines for a load of 1000#. In general the maximum secondary stresses occur at the points of primary stresses. Except for a couple of the members near the center of the truss the secondary stress for loads to the right of the center is small and might well be neglected.

In the next problem tried the bridge was assumed as hinged at the center, that $M_{68}=M_{78}=M_{79}=0$. The method of solution was exactly similar to the one previously used. Table X gives a comparison of the stresses occuring and the percentage of difference between the two cases. For stress near the end which is really the most important the error is very small.

In case only one or two members wished to be investigated it is possible to cut the bridge, as shown on sheet No. D, at joint 4 and 5 and consider only that portion to the left of the section. For members aB and ab this gives approximate results about 30% away from the trus stress.

In the next problem tried the structure was transformed into a Warren type by changing the inclined members Cd and dE to Dc and De (see sheet No. D). In this case the method used in solving for the secondary stress was similar as



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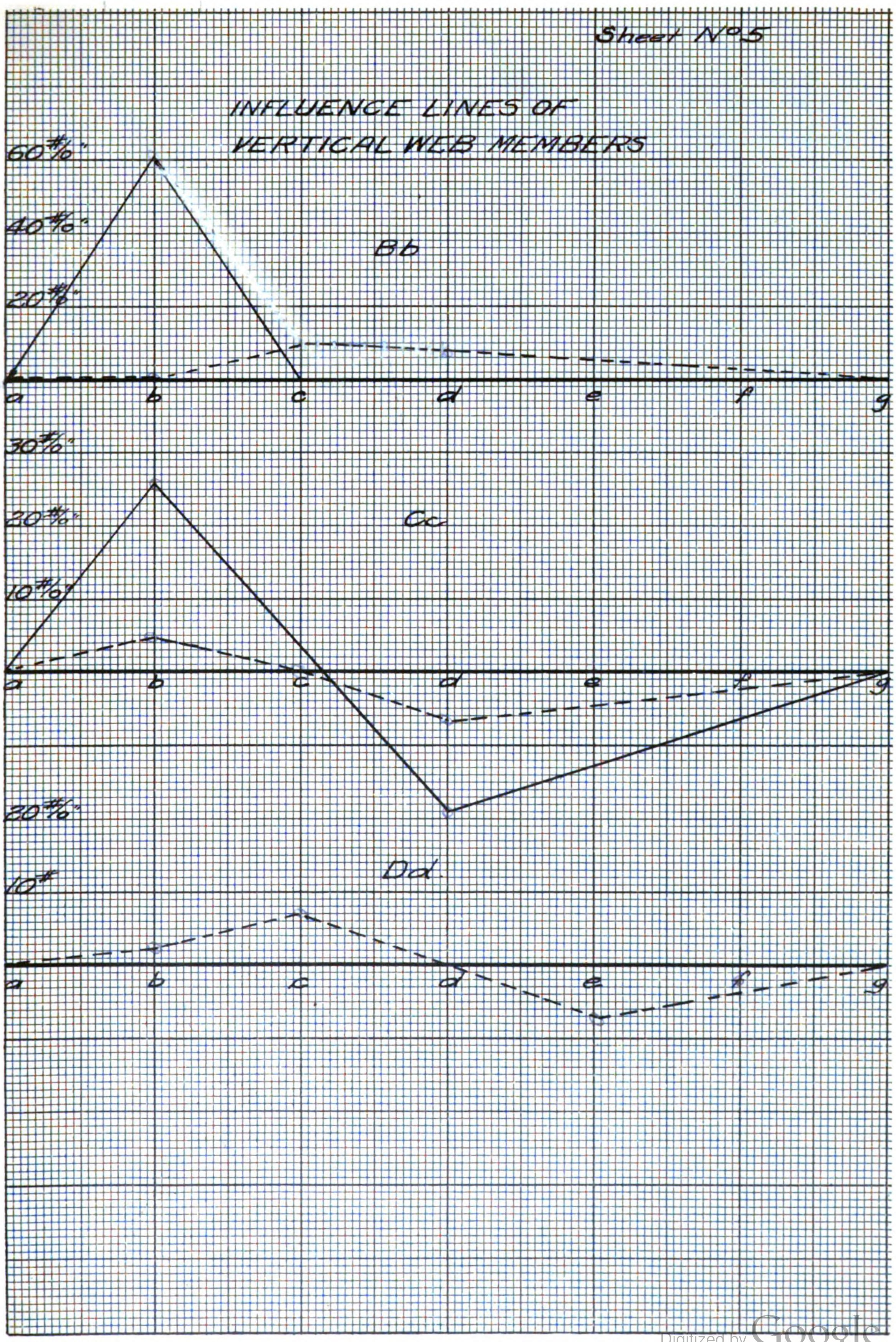

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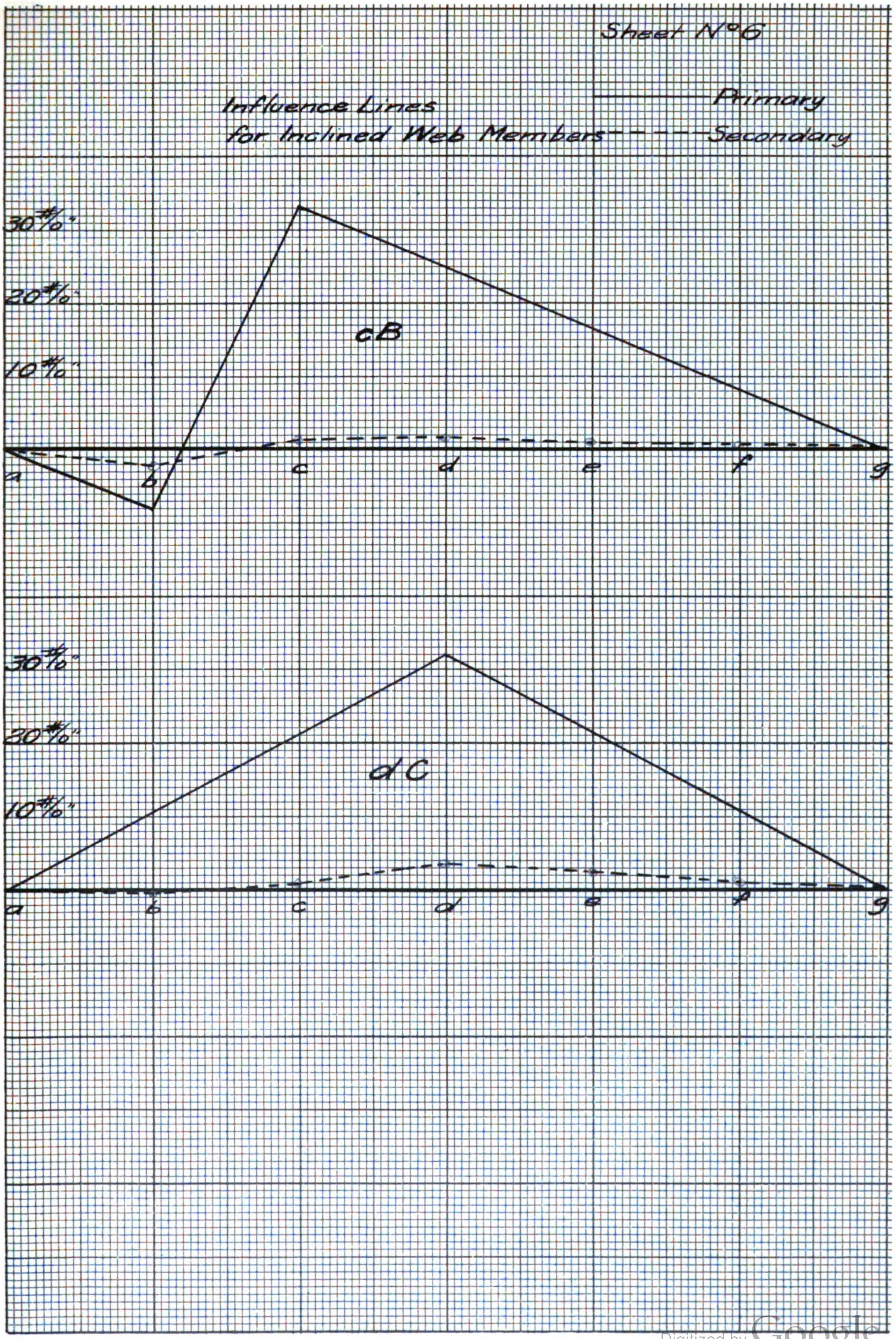

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B

BC

CD

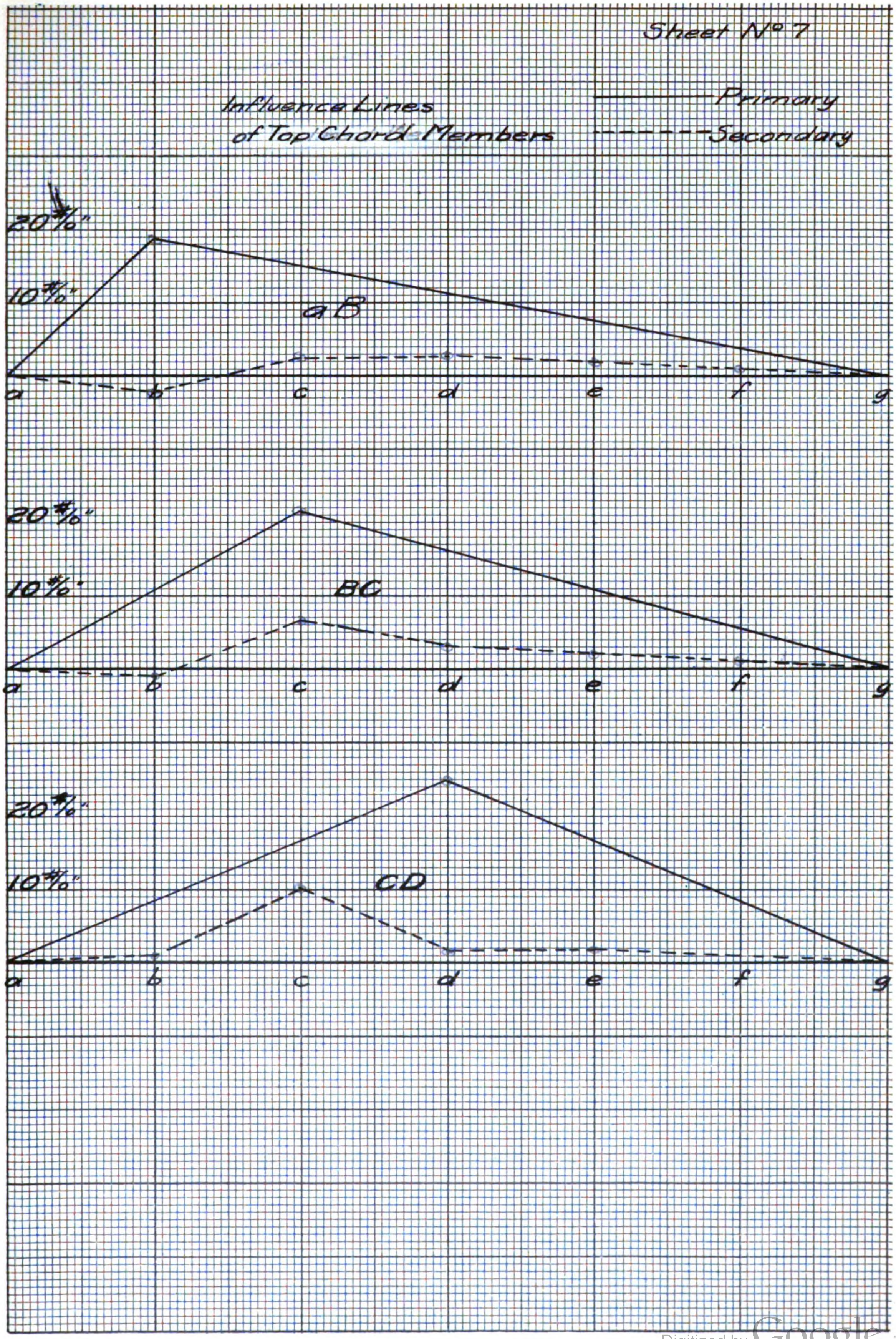
Sheet N° 7
Influence Lines
of Top Chord Members
Primary
Secondary
20#/0"
10#/0"
aB
a b c d e f g
20#/0"
10#/0"
BC
a b c d e f g
20#/0"
10#/0"
CD
a b c d e f g

TABLE IX.

ab	2320	23.10%	1260	5450	23.1%	1800	7770	23.1%
aB	1890	8.7%	385	4630	8.7%	550	6520	8.7%
Bb	1455	33.0%	1120	6280	33.0%	1600	7735	33.0%
bc	2570	17.5%	1050	6550	17.5%	1500	9120	17.5%
cB	2690	4.0%	345	7520	4.6%	455	10210	4.4%
BC	2310	17.1%	926	5420	17.1%	1320	7730	17.1%
Cc	980	46.0%	1525	3210	47.5%	1975	4190	47.1%
cd	2680	20.0%	1260	6250	20.0%	1800	8930	20.0%
dC	890	23.4%	576	5470	10.5%	785	6360	17.3%
CD	2550	10.2%	610	6010	10.2%	870	8560	10.2%
Dd	816	0		0	0	0	816	0

TABLE X.

Member	Bridge lingal at center.	Total length considered	Percentage of error.
1	-14.4	-13.7	5%
2	-15.0	-14.9	0%
3	+4.3	+4.2	2%
4	+4.6	+5.2	13%
5	-12.2	+12.7	4%
6	-13.4	+13.9	3.7%
7	-12.6	+12.1	4%
8	-0.5	-3.3	500%
9	+0.0	+0.8	
10	+3.0	+2.7	10%
11	+6.6	+ 7.0	6%
12	+11.1	-12.5	13%
13	+12.6	+10.2	19%
14	+9.8	+6.9	29%
15	-4.4	-8.9	100%
16	-15.0	-3.5	400%
17	+1.6	+4.0	200%
18	+5.8	+7.8	35%
19	+7.3	+9.1	24%
20	+7.2	-1.1	500%
21	0	+9.0	
22	0	+11.0	

explained on pages 5-7. Loads were applied as in the Pratt type of structure. Tables XI, XII, XIII give the solution of the problem with the load at "7" and table XIV gives the stress caused by a 1000# at each of the panel points. Table XV gives the relationship of the primary to the secondary stress.

In a pin connected bridge the top chord is usually made continuous. Assuming that the members were free to turn on their pins the secondary stresses were computed in the top chord. The work was greatly reduced as only the top chord was distorted and τ_2 τ_4 and τ_6 were the only unknowns.

A uniform load of 119,000# per panel was taken and by means of the first method, the secondary stress computed. In members 42 and 46 there was a secondary stress of 12.2 and 17% respectively, while in the rest of the members the secondary stress could be neglected. By placing the pins in the top chord with an eccentricity of 1" the stress in these two members to 20 and 249 respectively.

It is sometimes customary to place a collision strut in the end panel of a bridge and a problem was worked out with one in place. Without the strut the percent of secondary stress amounted to 10% of the primary stress. With the strut this stress was increased to 21%. The strut greatly shortens the length of the member and therefore increases the I/l for that member.

TABLE XI.

Triangle.	Angle	Value
	213	+13.0
A	321	+22.2
	231	-35.2
	325	-9.2
B	253	-29.4
	532	+38.6
	245	+65.3
C	452	-35.9
	524	-29.4
	465	+37.2
D	654	+13.4
	546	-50.6
	675	-168.1
E	756	+116.1
	567	+52.0

TABLE XII.

Joint	Angle	$\Delta\angle$	$\Sigma\Delta\angle$ $\tau=\tau_1+\Sigma\Delta L$	Members	I/1=K	$K \times \Sigma\Delta L$	load at 7 values of	load at 9
1	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
				12	9.151		-3.51	0.5
1	213	+13.0	+13.0	13	3.805	+49.5	+9.49	+13.5
					12.956			
	425	-29.4	-29.4	24	12.431		+11.17	+4.3
2				25	0.283	-8 .32	-18.2	-25.1
	523	-9.2	-38.6					
				23	0.255	-9.86	-27.4	-34.3
	321	+22.2	-16.4					
				21	9.151	150.00	-5.2	-12.1
					22.120			
				31	3.805	10.41	+10.41	-0.9
	132	-35.2	-35.2					
3				32	0.255	-8.97	-24.8	-36.1
	235	+38.6	+3.40					
				35	3.805	+12.92	+13.8	+2.5
					7.865	+3.95		
				46	12.431		-21.42	-2.0
	645	-50.6	-50.6					
				45	2.016	-102 .0	-72.0	-52.6
4	542	+65.3	+14.7					
				42	12.431	+183.0	-6.7	+12.7
					26.878	+81.0		
				53	3.805		-52.83	-10.1
	352	-29.4	-29.4					
				52	0.283	-8.32	-82.2	-39.5

TABLE XII CONT'D.

Joint	Angle	ΔL	$\Sigma\Delta L$ $\tau = \tau_1 + 2SL$	Members	I/l=k	$K \times \Sigma\Delta L$	load at 7 values of	load at 9
1	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
	254	-35.9	-65.3					
5				54	2.016	-131.50	-118.1	-75.4
	456	+13.4	-51.9					
				56	0.731	-37.90	-104.7	-63.0
	657	+116.1	+64.2					
				57	<u>5.959</u>	+<u>382.00</u>	+11.4	+54.1
					12.794	+204.28		
				68	12.431		-89.2	
	869	+37.2	+37.2					
				69	0.731	+27.2	-52.0	
	967	+52.0	+89.2					
6				67	0.774	+69.0	+ 0.0	
	765	+52.0	+141.2					
				65	0.731	+103.2	+52.0	
	564	+37.2	+178.4					
				64	<u>12.431</u>	+<u>2215.</u>	+89.2	
					27.098	+2414.4		
				75	5.959		+168.1	
	576	-168.1	-168.1					
7				76	0.774	-130.0	-0.0	
	679	+168.1	-336.2					
				79	<u>5.959</u>	<u>-2005.0</u>	-168.1	
					12.692	-2135.0		

Table XIII.

Joint I $25.912\,T_1 + 3.803\,T_3 + 9.151\,T_2 - 51.0 = 0$

II $44.240\,T_2 + 2.55\,T_3 + 9.151\,T_1 + 12.431\,T_4 + 2.83\,T_5 - 170.65 = 0$

III $15.730\,T_3 + 3.805\,T_1 + 2.55\,T_2 + 3.806\,T_5 + 47.54 =$

IV $53.756\,T_4 + 12.431\,T_2 + 2.016\,T_5 + 12.431\,T_6 + 2245.5$

V $25.558\,T_5 + 3.805\,T_3 + .283\,T_2 + 2.016\,T_4 + .731\,T_6 + 5.959\,T_7 + 414.4$

VI $\frac{54.196}{2}\,T_6 + 2414.4 = 0 \qquad T_6 = -44.55 \times 2 = -89.10$

VII $\frac{25.384}{2}\,T_7 - 2135 = 0 \qquad T_7 = +84.11 \times 2 = +168.22$

TABLE XIII.

Work Indicated	$\sqrt{}_{11}$	$\sqrt{}_{12}$	$\sqrt{}_{13}$	$\sqrt{}_{14}$	$\sqrt{}_{15}$	Abs.term
(1) ÷ 25.912 = (a)	1	.353	1.47			-1.97
(2) ÷ 9.151 = (b)	1	4.820	.028	1.360	.031	-18.65
(3) ÷ 3.805 = (c)	1	.067	4.140		1	12.50
(b - a)		4.467	-119	1.360	.031	-16.68
c - a		-.286	3.993			+14.47
(b-a)÷4.467 = (d)		1	-0.0266	.304	.0069	-3.73
(c-a)÷ 286 = (e)		1	-13.960		-3.500	-50.59
(4) ÷ 12.431 = (f)		1		4.320	.162	+91.20
5 ÷ 283 = (g)		1	13.440	7.130	90.417	+4775.54
(e - d)			-13.953	-.304	-3.507	-46.86
(f - d)			+.027	+4.016	+.155	+93.93
(g - d)			13.467	6.826	90.410	+4779.27
(h) (e-d)÷-13.933			1	.022	.252	+3.37
(f-d)÷.027 (h)			1	148.741	5.740	+3478.88
(g-d)÷13.467 (j)			1	.507	6.710	+354.88
(i - h)				148.740	5.488	+3475.51

TABLE XIII CONT'D.

Work Indicated		$\sqrt{1_1}$	$\sqrt{1_2}$	$\sqrt{1_3}$	$\sqrt{1_4}$	$\sqrt{1_5}$	Abs.term
(j - h)					.485	6.458	+351.51
(i-h)+148.74	(k)				1	.037	+23.37
(j-k)+ 485	(l)				1	13.315	+724. 76
l - h						13.278	+701.39
Value		-3.51	11.17	10.41	-21.42	-52.83	

TABLE XIV.

Member	1000# L. Stress due to load at 3	1000# L Stress due to load at 5	1000#PL Stress due to load at 7	1000#P L stress load at 9	1000#P L stress load at 11
12	-1.130	+.798	-.700	-.430	-.230
13	+5.08	-.380	+1.674	+.990	+.546
24	-4.24	+4.26	+2.62	+1.282	+.720
25	-.440	-.914	-1.922	-.964	-.564
23	-.804	-5.800	-2.290	-1.01	-.832
21	+3.840	-2.330	-.794	-.910	-.492
31	+13.70	-2.520	+1.726	+.444	+.314
32	-.478	-6.100	-2.220	-1.050	-.978
35	-14.42	+2.23	-1.434	-.188	-.210
46	-9.85	-11.22	+2.66	-.026	-.314
45	-.220	-1.344	-10.56	-4.94	-2.03
42	-1.276	+11.74	+22.10	+1.79	+1.154
53	-9.820	-8.280	-5.22	-.666	-.712
52	+.956	-.668	-2.96	-1.122	-.688
54	+3.60	-.800	-12.360	-5.45	-2.57
56	+2.89	-.560	-3.84	-2.61	-1.60
57	+3.580	-3.28	+10.86	+3.96	+2.44
67	+2.69	+9.96	00.00	-9.96	-2.69
65	+2.66	+1.74	00.00	-2.36	-1.414
64	-1.226	-.740	+6.10	-.188	-.454
75	+1.966	+.360	+10.22	+1.760	+2.440
76	+2.090	+5.84	00.00	-5.84	-2.090

TABLE XV.

Members		35750 #/PL DL Stress Secondary	DL Stress Direct	% sec.stress to Direct stress	83800#PL L.L.Stress Secondary
ab	13 31	244	2320	10.5%	571
aB	12 21	48.1	1890	2.5%	112
Bb	23 32	212	1455	14.5%	496
bc	35 53	444	2570	17.3%	1040
cB	25 52	85.1	2690	3.2%	228
BC	24 42	653	2310	28.2%	1487
Cc	45 54	341	0	—	800
cd	57 75	307	2680	11.4%	702
cD	56 65	102	890	11.5%	337
CD	46 64	367#	2550	14.4%	858#
Dd	67 76	0	2430	0	307# 1/2 bridge loaded.

Members		83800#PL LL Stress Direct	% sec. stress to Direct stress	DL+LL Secondary	DL+LL Direct	% Secondary stress to Direct stress.
ab	13 31	5450	10.5%	815	7770	10.5%
aB	12 21	4630	2.5%	160	6250	2.5%
Bb	23 32	6280	14.5%	708	7735	14.5%
bc	35 53	6550	17.3%	1484#	9120	17.3%
cB	25 52	7520	3.0%	313	10210	3.1%
BC	24 42	5420	28.2%	2140	7730	28.2%
Cc	45 54	0	---	1141	0	----
cd	57 75	6250	11.4%	1009	8930	11.4%
cD	56 65	5470	6.2%	439	6360	6.9%
CD	46 64	6010	14.4%	1225	8560	14.4%
Dd	67 76	5700	--	--	8130	---

From the results obtained it is evident that of the two structures investigated the Warren type is the more desirable if secondary stresses are to be considered and from the percentage of secondary stresses to primary stresses it is shown that the secondary stresses should not be disregarded. The stresses are found to be the greatest near the supports of the truss and to depend upon the value of I/l. For small secondary stress therefore, the panels should be long and the volume of material be a minimum so as to keep the moment of inertia low. The rigid members, such as the top chord members, suffer more than the flexible eyebars.

APPROVED BY F. E. Furneaux

Professor in charge.